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TECH. NOTE G.W.394

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ROYAL AIRCRAFT ESTABLISHMENT

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A STUDY OF AZIMUTH GUIDANCE AND CONTROL FOR THE VERNIER STAGE OF BLUE STREAK

by

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A Study of Azimuth Guidance and Control for the Vernier Stage of Blue Streak

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SUMMARY

A control system in azimuth based on a small swivelling jet is proposed for the vernier stage of the medium range ballistic missile Blue Streak. This control system is applicable to both inertia and radar guidance systems, the measured error quantity being essentially the azimuth component of the deviation of the missile flight path from the line joining missile and target.

The jet has to be sufficiently small to avoid a missile speed error at cut-off. This leads to a relatively small available deflecting force in azimuth and, to make full use of this, a system with a high loop gain and limited error input is proposed, stabilization being obtained with heading and heading rate feedback. The input limits and gains are chosen so that, in effect, the missile heading is limited within prescribed bounds.

The performance of the system has been studied on an analogue simulator under assumptions of constant missile parameters and the usual small angle approximations. It is shown that, with proper choice of control coefficients, the system is very little worse than an ideal, infinitely stiff, system with the same heading limitation.

The steady state error due to heading gyro wander is considered, and for the missile parameters used is thought to be tolerable.

The effects of the data sampling and computer time lag relevant to the radar guidance system have been studied and it is shown that sampling periods and time lags up to about 1 second have no material effect on the control system.

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1 Introduction

The controlled flight of the medium range ballistic missile, Blue Streak, can be divided into two propulsion stages. During the first or main propulsion stage, the missile is boosted to a velocity of about 18,000 ft/sec at a height of some 100 n. miles. The overall noise in the control system during this stage may be too large for (a) sufficiently accurate azimuth control, and (b) sufficiently accurate cut-off timing. Provision is therefore being made for a second propulsion stage, the vernier stage, using small auxiliary motors. During this stage the noise level will be lower and the problems of removing the residual azimuth errors and effecting the correct "cut-off" are simplified.

Control of the missile throughout the propulsion stages is obtained solely by swivelling the jets. During the main stage the control problem is complicated by the rapidly varying conditions of speed, mass, and aerodynamic forces on the missile. During the vernier stage, however, which is virtually in vacuo, the aerodynamic forces are negligible and the variations in speed and mass of the missile are very small. For this reason, the problem of azimuth control during the vernier stage has been tackled first, and this note deals with the design and simulator study of a suitable control system.

As will be shown, this study is relevant to both inertia and radar guidance systems. In the case of the radar system, further work is included on the effect of computer lags and the intermittent nature of the error signal.

2 Equations of motion

In Fig.1 Ox and Oy are taken as reference space axes in the azimuth plane. The axis of the missile makes an angle ψ (heading angle) with Ox, and the thrust, P, acting at a distance ℓ to the rear of the centre of gravity, makes an angle ζ with the missile axis. The tangent to the flight path makes an angle ψ_m with Ox and the target direction makes an angle ψ_T with Ox. Let m be the mass of the missile, B its lateral moment of inertia and v the magnitude of its velocity component in the azimuth plane.

The equations of motion then are:-

$$m\ddot{x} = P \cos(\psi + \zeta) \qquad (1)$$

$$m\ddot{y} = P \sin(\psi + \zeta) \qquad (2)$$

$$B\ddot{\psi} = -P\ell \sin \zeta \qquad (3)$$

$$\dot{x} = v \cos \psi_m \qquad (4)$$

$$\dot{y} = -v \sin \psi_m . \qquad (5)$$

These can be reduced by eliminating x and y. Differentiating equations (4) and (5) and substituting for $\ddot{x}$, $\ddot{y}$, in equations (1) and (2):-

$$m(\dot{v} \cos \psi_m - v\dot{\psi}_m \sin \psi_m) = P \cos(\psi + \zeta) \qquad (6)$$

$$m(\dot{v} \sin \psi_m + v\dot{\psi}_m \cos \psi_m) = P \sin(\psi + \zeta) \qquad (7)$$

whence $$\dot{v} = \frac{P}{m} \cos (\psi+\zeta-\psi_m) \tag{8}$$

and $$\dot{\psi}_m = \frac{P}{mv} \sin (\psi+\zeta-\psi_m) \tag{9}$$

and from equation (3),

$$\ddot{\psi} = -\frac{P\ell}{B} \sin \zeta . \tag{10}$$

Equations (8) to (10) describe the motion of the missile in response to a given jet deflection ζ. However, they are unnecessarily complicated for the order of investigation initially required. The following assumptions are therefore made:-

(i) ζ and $(\psi-\psi_m)$ are sufficiently small for $\sin \zeta$, $\sin (\psi+\zeta-\psi_m)$ to be replaced by ζ, $(\psi+\zeta-\psi_m)$.

(ii) $\frac{P}{mv}$ and $\frac{P\ell}{B}$ are constant.

(From equation (8) and the small angle approximation, $\frac{P}{m} \doteq \dot{v}$, so that the former assumption is equivalent to $\frac{\dot{v}}{v}$ = constant.)

With these assumptions, the equations of motion reduce to:-

$$\boxed{\begin{aligned} \dot{\psi}_m &= \frac{1}{T_1} (\psi+\zeta-\psi_m) \\ \ddot{\psi} &= -w_o^{\,2} . \zeta \end{aligned}} \qquad \begin{aligned} &(11) \\ &(12) \end{aligned}$$

where T_1 is measured in seconds and w_o is measured in radians/second, and

$$\left\{ \begin{aligned} \frac{1}{T_1} &= \frac{P}{mv} \\ w_o^{\,2} &= \frac{P\ell}{B} . \end{aligned} \right.$$

For the present study the values $T_1 = 2000$ secs, $w_o = 1$ rad/sec, were taken. These could represent, for example, a missile with a thrust-to-mass ratio of $\frac{1}{4}$g, a speed of 16,000 ft/sec, and a moment of inertia (taking $B \doteq \frac{1}{3} m\ell^2$) corresponding to a uniform thin rod of half-length $\ell = 24$ ft.

3 Guidance and control equations

In the case of the inertia guidance system and one possible method of radar guidance computation, the measured error quantity is the line error, ΔL, at the target, assuming the missile were to continue along its present flight path (Fig. 2).

Thus,

$$\Delta L \doteqdot R\,(\psi_m - \psi_T)$$

where R is the range to go.

Now R changes by perhaps 100 miles during the vernier stage, that is to say by perhaps 5% of its value at out-off. For the present purpose it can therefore be considered constant, and $\psi_m - \psi_T$ is proportional to the line error ΔL.

In the case of another proposed method of radar guidance computation, the azimuth error is measured as the lateral velocity component perpendicular to the desired velocity direction, that is to say, $v \sin(\psi_m - \psi_T)$. But, since v may be considered constant and $\sin(\psi_m - \psi_T) \doteqdot \psi_m - \psi_T$, it follows that $\psi_m - \psi_T$ is again an adequate representation of the error measurement.

Thus the two methods of guidance in azimuth are identical; in the inertia case, however, the estimation of the line error ΔL is possible only if the missile is close to a predetermined trajectory, while in the radar system there is no such restriction.

In practice, the measured value of $(\psi_m - \psi_T)$ will lag the true value, either accidentally, because of instrument lags, or purposefully, to introduce smoothing.

Let ψ'_ε be the output of the error measuring system, i.e. the guidance system, and assume that this system has a linear lag with time constant T_o.

The guidance equation then takes the form:-

$$\boxed{\begin{aligned} \psi'_\varepsilon + T_o \frac{d\psi'_\varepsilon}{dt} &= \psi_T - \psi_m \\ &= \psi_\varepsilon, \text{ say.} \end{aligned}} \qquad (13)$$

In the case of a ground computer for measuring $(\psi_T - \psi_m)$ this is not a realistic approach, but as already mentioned, the effect of finite time delays is considered in a later section.

The proposed control equation, in general terms, is:-

$$\boxed{\zeta_D = -K_1 Y_1(D) \cdot \psi'_\varepsilon + K_2 \dot{\psi} + K_3 \psi} \qquad (14)$$

in the usual notation.

Here ψ and its derivative, are assumed to be measured by heading and/or rate gyros in the missile. The bandwidth of the motor-swivelling servos is assumed to extend beyond the highest frequencies likely to be encountered in the missile behaviour. This is reasonable since the system frequencies are relatively low.

A simple block diagram of the complete system is shown in Fig.3, and a representation of the linearized equations in Fig.4.

4 Limit on missile heading angle

In this note the term "heading angle" is occasionally used in contexts where it might be thought preferable to use the term directional error, the latter term being defined, in this context, as $(\psi-\psi_T)$. However, if it is assumed that the heading gyro datum is parallel to the target direction (within a few degrees) then the heading angle, ψ, as measured by the gyro, is a sufficiently good estimate of the directional error, $(\psi-\psi_T)$. In a radar guided system it is desirable to limit the directional error so that its magnitude does not exceed say 30°. However, in the simulation of such a system, with $\psi_T = 0$, it is the heading angle, ψ, which is limited, so that this is the situation treated in the analysis.

5 Preliminary analysis - the ideal system

The main object of the vernier stage (apart from cut-off adjustment) is to eliminate azimuthal errors as quickly as possible, at the same time avoiding large heading and jet angles. This means that the control system will be highly non-linear, and a linear analysis is not sufficient.

Some insight into the problem may be obtained by considering a system which we may call the ideal one. Let us suppose that ψ is limited in magnitude to the value ψ_{max}. Then in order to obtain the best response to a step in ψ_T (which is equivalent to having an initial lateral velocity error), the best that can be done is to put on the maximum heading angle as rapidly as possible in order to obtain the maximum lateral acceleration, hold this heading until the lateral velocity is correct (i.e. $\psi_\varepsilon = 0$), and then reduce the heading rapidly to a small value. This is illustrated in Fig.5.

Since ψ starts ideally with a step, ζ (proportional to $\ddot{\psi}$) is a second-order δ-function. The angular error, ψ_ε, ideally decreases linearly to zero, for when $\psi = \psi_{max}$, $\zeta = 0$, $\psi_m << \psi$ and therefore $\dot{\psi}_m \doteqdot 1/T_1 \cdot \psi_{max}$ from equation (11); hence, from equation (13), since ψ_T is constant,

$$\dot{\psi}_\varepsilon \doteqdot -\frac{1}{T_1}\psi_{max} = -\frac{P}{mv}\psi_{max} . \qquad (15)$$

The rate of reduction of error is thus approximately proportional to the maximum permissible heading-to-flight path angle, and to the thrust-to-mass ratio.

It will be seen later that equation (15) is very nearly realised in practice.

6 The control equation

In order to achieve a system which approaches the ideal one, we are led to a control equation involving heading and rate of change of heading, together with a very high loop gain in the guidance loop and a limit on the guidance input.

Thus:-

$$\zeta_D = -K_1Y_1(D)\,\psi'_\varepsilon + K_2\dot{\psi} + K_3\psi \qquad (16a)$$

$$\text{for} \quad |Y_1(D)\,\psi'_\varepsilon| < A$$

and

$$\zeta_D = \mp K_1A + K_2\dot{\psi} + K_3\psi \qquad (16b)$$

$$\text{for} \quad |Y_1(D)\,\psi'_\varepsilon| > A .$$

ζ_D (the demanded jet angle) is the input to the jet servos; if it is assumed that the achieved jet angle is limited by mechanical stops to $\pm\zeta_L$, then

$$\zeta = \zeta_D \qquad \text{for} \qquad |\zeta_D| < \zeta_L \tag{17a}$$

$$\text{and} \qquad \zeta = \pm\zeta_L \qquad \text{for} \qquad |\zeta_D| > \zeta_L \tag{17b}$$

neglecting the servo lags, as stated previously.

7 Preliminary choice of parameters

The stability of the system must be considered in the two separate cases of settling to the maximum heading angle (the 'non-linear' settling), and the final linear settling.

Considering the settling to the maximum heading angle, when the jet deflection is off the limits but $|Y_1(D)\ \psi'_\varepsilon| > A$, we have

$$-\frac{\ddot{\psi}}{w_o^2} = \mp K_1 A + K_2\dot{\psi} + K_3\psi \tag{18}$$

which is a linear differential equation in ψ, with the steady state heading angle, $\psi_s = \pm\frac{K_1 A}{K_3}$, undamped angular frequency $w = w_o\, K_3^{\frac{1}{2}}$, and damping ratio, $u = \frac{\frac{1}{2}K_2 w_o}{K_3^{\frac{1}{2}}}$.

Now we require a steady state $|\psi_s| = \psi_{max}$ and a damping ratio of 0.8, say, which gives the relations:-

$$\frac{K_1 A}{K_3} = \psi_{max} \tag{19}$$

$$\text{and} \qquad K_2 = \frac{1.6\, K_3^{\frac{1}{2}}}{w_o} = \frac{1.6 w}{w_o^2}. \tag{20}$$

Consider now the final stage, taking $Y_1(D) = 1$.

The equations are

$$\left.\begin{aligned} \dot{\psi}_m &= \frac{1}{T_1}(\psi+\zeta-\psi_m) \\ \ddot{\psi} &= -w_o^2\zeta \\ \text{and} \qquad \zeta &= -K_1(\psi_T-\psi_m) + K_2\dot{\psi} + K_3\psi\ . \end{aligned}\right\} \tag{21}$$

Eliminating ζ and ψ_m one obtains

$$\left[\left(D + \frac{1}{T_1}\right)\left(D^2 + w_o^2 K_2 D + w_o^2 K_3\right) + \frac{K_1}{T_1}\left(w_o^2 - D^2\right)\right]\psi = K_1 w_o^2 \left(D + \frac{1}{T_1}\right)\psi_T ,$$

where $D \equiv \frac{d}{dt}$.

The stability polynomial is thus a cubic:-

$$p^3 + \left(w_o^2 K_2 + \frac{1}{T_1} - \frac{K_1}{T_1}\right)p^2 + \left(\frac{K_2 w_o^2}{T_1} + K_3 w_o^2\right)p + \left(\frac{K_3 w_o^2}{T_1} + \frac{K_1 w_o^2}{T_1}\right).$$

Since $K_1 >> 1$, $T_1 >> 1$, the terms $\frac{1}{T_1}p^2$, $\frac{K_2 w_o^2}{T_1}p$, and $\frac{K_3 w_o^2}{T_1}$, can be neglected for the present purpose, leaving the simpler form:-

$$p^3 + \left(w_o^2 K_2 - \frac{K_1}{T_1}\right)p^2 + K_3 w_o^2 p + \frac{K_1 w_o^2}{T_1} . \tag{22}$$

Finally, making the substitutions $K_3 = \frac{w^2}{w_o^2}$, $K_2 = \frac{1.6w}{w_o^2}$ (i.e. taking the damping ratio of the non-linear settling to be 0.8, as above), the stability cubic becomes

$$\boxed{p^3 + \left(1.6w - \frac{K_1}{T_1}\right)p^2 + w^2 p + \frac{K_1 w_o^2}{T_1}} . \tag{23}$$

This cubic can be written in the form:-

$$(p^2 + 2\alpha p + \beta)(p + \gamma)$$

and the roots were first investigated for the case $w_o = 1$ rad/sec, $w = 1$ rad/sec, $T_1 = 2000$ secs and various values of K_1; the results are shown in Figs. 6 and 7.

It will be seen that the decay times of the two modes are equal when $K_1 = 540$. In this case the damping ratio of the oscillatory mode is about 0.58, which is satisfactory.

The value of w was chosen to be equal to w_o in this analysis, which is equivalent to an initial demand for jet angle from the guidance system just equal to the limiting value, ζ_L. The effect of varying the natural frequency, w, of the autopilot loop is considered in section 11.

8 The simulator arrangement

Initially a laboratory-built simulator was used, but at a later stage the work was transferred to a G-PAC simulator. The equations set up on the simulator were equations (11), (12), (13), (16) and (17), the shaping network in equation (16) being a simple phase advance:-

$$Y_1(p) = \frac{1 + pT_A}{1 + apT_A} .$$

A block schematic of the simulator arrangement is shown in Fig.8.

It was soon found that, contrary to normal experience, the presence of phase advance had a destabilizing effect on the system for the values of gain used. The phase advance network was therefore by-passed, and with the parameter values

$$\begin{cases} w_0 = 1 \text{ rad/sec}; \quad T_1 = 2000 \text{ sec}; \quad T_0 = \frac{1}{4} \text{ sec}; \\ K_3 = 1; \quad K_2 = 1.6; \quad K_1 = 550; \quad K_1 A = \frac{1}{2} \end{cases}$$

the simulator records shown in Fig.9 were obtained in response to an initial step in ψ_T of magnitude 0.003 rad.

These results confirm the rough paper analysis given above, and show that the system can be satisfactorily damped.

Further records, all in response to a 0.003 rad step in ψ_T, showing the effect of different initial conditions, are given in Fig.10. The control parameters have the same values as given above.

In Appendix I the system is considered from the frequency response point of view, and an analysis of the effect of phase advance is given.

9 The time taken to reduce an initial error

In the ideal system (section 4 and Fig.5) an initial flight path error is reduced linearly with time, and from equation (15) it is seen that the time required to reduce to zero an initial error, ψ_ε, is

$$t = \frac{|\psi_\varepsilon| \, T_1}{\dot{\psi}_{max}} . \tag{24}$$

In practice with $T_1 = 2000$ secs and using the "optimum" damping arrangement as above, it was found from the simulator work that the time required is about 4 secs more than that given by equation (24).

The simulator results are compared with those for the ideal system in Fig.11.

10 Steady state errors

The use of heading angle feedback in the inner loop means that steady state errors can occur due to (i) changes in the target direction, and (ii) gyro drift.

Let ψ' be the measured value of ψ which replaces ψ in equations (16), and let $\psi' - \psi = \Delta\psi$, the gyro drift angle. Then from equations (11), (12), (13) and (16), using the suffix s to denote steady state conditions,

$$\begin{aligned} \zeta_s &= 0 \\ \dot{\psi}_s &= \dot{\psi}_{ms} \\ \psi_{Ts} - \psi_{ms} &= \psi_{\varepsilon s} = \psi'_{\varepsilon s} \\ \zeta_s &= -K_1 \psi'_{\varepsilon s} + K_3 \psi'_s . \end{aligned}$$

It follows that the steady state flight path error is

$$\psi_{\varepsilon s} = \frac{K_3}{K_1+K_3}\left(\psi_{Ts} + \Delta\psi_s\right) . \qquad (25)$$

Thus if the gyro axis is along the assumed sight-line at launch, ψ_{Ts} represents the change in the sight-line during the controlled flight and $\Delta\psi_s$ represents the steady gyro drift. In theory a correction could be made for the first part of the error, but in practice the change in the sight-line is likely to be, at most, a few minutes of arc, whereas the gyro error may be a few degrees.

Also, $K_1 >> K_3$ and so, approximately,

$$\psi_{\varepsilon s} \doteqdot \left(\frac{K_3}{K_1}\right)\Delta\psi_s .$$

From Fig.2, the line error due to this is

$$\Delta L \doteqdot R \cdot \psi_{\varepsilon s} \doteqdot \left(\frac{K_3}{K_1}\right) R \cdot \Delta\psi_s \qquad (26)$$

where R is the range from cut-off to impact.

For example, if $K_3 = 1$, $K_1 = 500$, $R = 2000$ n. miles, then a gyro drift of 7° would cause a line error, ΔL, of about $\frac{1}{2}$ n. mile.

The factor $\frac{K_3}{K_1}$ will be called the DRIFT FACTOR.

11 <u>The inner loop gain</u>

In the work described so far the inner loop gain, K_3, has been arbitrarily fixed so that w, the natural angular frequency of the autopilot loop, was equal to w_o. The effect of varying K_3 can be studied by considering the stability cubic equation (23) again. If we assume, as in section 7, that a suitable criterion for the choice of K_1 is equal distribution of the total damping between the two modes, i.e. $\alpha = \gamma$, this leads to a determinate value of K_1. The equations to be solved are

$$\left.\begin{aligned} 3\alpha &= 1.6w - \frac{K_1}{T_1} \\ \beta + 2\alpha^2 &= w^2 \\ \beta\alpha &= \frac{K_1}{T_1} \cdot w_o^{\,2} . \end{aligned}\right\} \qquad (27)$$

These have been solved for various values of w, with $w_o = 1$ rad/sec and $T_1 = 2000$ secs, the results being shown in Figs. 12, 13 and 14.

It will be seen that the available damping is virtually independent of w, but that, as w is increased, the gain K_1 and the angular frequency, $\sqrt{\beta}$, of the quadratic mode increase, but the damping ratio, $\frac{\alpha}{\sqrt{\beta}}$, decreases. The drift factor improves as w increases.

The effect of increased inner loop gain on the overall system was studied on the simulator, and a simulator record is shown in Fig.15 for the case $K_3 = 3$, $K_2 = 3.46$, $K_1 = 2,800$. It will be seen that there is no improvement on the previous records, so that the only advantage of a higher gain appears to be some improvement in the drift factor. A disadvantage of a high gain which has not yet been studied in detail is the effect of natural vibrations of the missile on the measurement of ψ and $\dot{\psi}$.

12 Intermittent guidance

In the case of the radar guidance system, the guidance information is not available continuously but only at specified intervals of time. The information may also be late to the extent of one time interval unless special prediction computation is done by the ground computer. The effect of this sampling and time delay has been studied theoretically and on the simulator, and details are given in Appendices II, III and IV. In this part of the study the thrust-to-mass ratio of the missile during the vernier stage was assumed to be $\frac{1}{8}$g rather than $\frac{1}{4}$g, the former being a more recent estimate of a realistic value for $\frac{P}{m}$. The main conclusion is that the proposed control system is satisfactory if the sampling interval and time delay are less than about 1 sec. This appears to be well within current estimates of the computer time delay.

13 Results and conclusions

(i) It has been shown by simulator work and paper analysis that a simple control system involving heading and heading rate feedback, and with limits on the azimuth angular error input, can be used for the control in azimuth during the vernier stage. A relatively high gain system is recommended in order to make full use of the available correcting thrust.

(ii) If K_1, K_3, K_2 are the coefficients of flight path error ψ_ε, heading angle ψ, and heading rate $\dot{\psi}$, respectively, the recommended values are $K_1 = 0.27\ T_1$ rads/rad, $K_2 = 1.6/w_o$ rads/rad/sec, $K_3 = 1$ rad/rad, where

$$\left.\begin{aligned} T_1 &= \frac{\text{missile momentum}}{\text{thrust}} \quad (\text{in secs}) \\ w_o^2 &= \frac{\text{thrust} \times \text{moment arm}}{\text{moment of inertia}} \quad (\text{in sec}^{-2}) \end{aligned}\right\} \text{ assumed constant.}$$

(iii) With the above control coefficients the time taken to reduce to zero an initial error, ψ_ε, in the flight path is

$$t \doteqdot \frac{\psi_\varepsilon T_1}{\psi_{max}} + (\delta t)$$

where ψ_{max} is the maximum allowable heading angle and δt is approximately 4 secs when $T_1 = 2000$ secs (see Fig.11).

(iv) The use of the heading term in the control equation leads to steady state flight path errors, and hence to a resultant line error, due to gyro drift. The resultant line error, ΔL, due to a gyro drift of $\Delta\psi_s$ is given by

$$\Delta L \doteqdot \left(\frac{K_3}{K_1}\right) R \cdot \Delta\psi_s$$

where R is the range from cut-off to impact.

Thus if $T_1 = 2000$ secs and $R = 2000$ n. miles, a gyro drift of 7^o causes a line error of about $\frac{1}{2}$ n. mile.

(v) From (ii), (iii) and (iv), it is seen that if the thrust-to-mass ratio is reduced, the time to correct an initial error will be longer, but the line error due to gyro drift will be less.

(vi) The effect of sampling the flight path error at intervals of τ secs and the effect of a finite delay, τ, in the correcting signal, have been studied for the case $T_1 = 4000$ secs, $w_o = \frac{1}{2}$ rad/sec (corresponding to the latest estimate of thrust-to-mass ratio of $g/8$, as mentioned in section 12). Provided the gain K_1 is slightly reduced below the value given by (ii) above, the sampling and time delay have virtually no effect up to $\tau = 1$ sec.

14 Further work

(i) It is proposed to extend the investigation of azimuth control to the main stage, where the variation of mass, speed, etc. cannot be neglected.

(ii) The effect of (a) flexural oscillations and (b) fuel sloshing on the control system will have to be studied.

(iii) Work is to be done on the effects on the control system of various kinds of "noise", and on the data smoothing required to minimise such effects.

REFERENCES

No.	Author	Title, etc
1	A.P. Roberts	Sampled Data Control Systems for Beam-Riding Missiles. RAE Tech Note No. GW 380.

Attached:- Appendices I to IV
Drgs. GW/P/6831 to 6845
Detachable Abstract Cards

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APPENDIX I

Frequency response analysis

Taking the Laplace transforms of equations (11), (12), (13) and (14),

$$\left.\begin{aligned} p\bar{\psi}_m &= \frac{1}{T_1}(\bar{\psi} + \bar{\zeta} - \bar{\psi}_m) \\ p^2\bar{\psi} &= -w_o^2\bar{\zeta} \\ (1+T_op)\bar{\psi}'_\varepsilon &= \bar{\psi}_\varepsilon \\ \bar{\zeta} &= -K_1Y_1(p)\bar{\psi}'_\varepsilon + (K_2p + K_3)\bar{\psi}. \end{aligned}\right\} \tag{28}$$

Eliminating $\bar{\psi}$ from the first two equations

$$\frac{\bar{\psi}_m}{\bar{\zeta}} = \frac{p^2 - w_o^2}{p^2(1+T_1p)}.$$

Hence, substituting for $\bar{\zeta}$, $\bar{\psi}$ in terms of $\bar{\psi}_m$ and for $\bar{\psi}'_\varepsilon$ in terms of $\bar{\psi}_\varepsilon$ in the final equation the open-loop transfer function is obtained, namely

$$\frac{\bar{\psi}_m}{\bar{\psi}_\varepsilon} = \frac{-K_1Y_1(p)(p^2-w_o^2)}{(1+T_op)(1+T_1p)(p^2+w_o^2K_2p+w_o^2K_3)} \tag{29}$$

$$= Y_L(p), \quad \text{say.}$$

With the figures given in the text, namely

$$w_o = 1, \quad T_1 = 2000, \quad T_o = \tfrac{1}{4}, \quad K_2 = 1.6, \quad K_3 = 1, \quad K_1 = 550$$

and with no phase advance, i.e. $Y_1(p) = 1$, this gives

$$Y_L(p) = \frac{550\,(1-p^2)}{(1+\frac{1}{4}p)(1+2000p)(p^2+1.6p+1)}.$$

The gain- and phase-log frequency curves for $Y_L(j\omega)$ have been computed and are shown in Fig. 16.

From these curves, it is seen that there is a phase margin of about 60°, which is ample for stability purposes.

In order to determine the effect of phase advance on the system, the characteristic polynomial was studied.

The characteristic polynomial is the denominator in the closed-loop transfer function $\dfrac{Y_L(p)}{[1+Y_L(p)]}$.

Putting

$$Y_1(p) = 1 + T_A p ,$$

in equation (29), the characteristic polynomial is found to be

$$Ap^4 + Bp^3 + Cp^2 + Dp + E$$

where

$$A = T_o T_1$$

$$B = T_o + T_1 + w_o^2 K_2 T_o T_1 - K_1 T_A$$

$$C = 1 + w_o^2 K_2 (T_o + T_1) + w_o^2 K_3 T_o T_1 - K_1$$

$$D = w_o^2 K_2 + w_o^2 K_3 (T_o + T_1) + K_1 T_A w_o^2$$

$$E = w_o^2 (K_3 + K_1) .$$

The Routh-Hurwitz stability criterion demands that, for stability

$$B > 0$$

$$BC-DA > 0$$

and

$$(BC-DA)\,D - B^2E > 0 .$$

With the values for w_o, T_1, T_o, K_2, K_3 given above the first condition becomes $K_1 T_A < 2800$. Thus, for a given K_1, a sufficient increase in the phase advance time constant causes instability.

For $K_1 = 1000$, the stability condition is found to be $T_A < 2.0$. This system was tested on the simulator and was found to go unstable at about $T_A = 1.3$. This discrepancy is due to the phase advance impurity which was not included in the above analysis.

APPENDIX II

Intermittent guidance data

II.1 Introduction

It has been shown that a suitable azimuth control system can be designed for the vernier stage, provided that the guidance information is available continuously, i.e. regarding the overall system as a continuous servo. In the radio system, however, the guidance signal is computed on the ground by a digital computer from data provided by a doppler-radar system. This means that the guidance signal is not a continuous function of time but may take only those values appropriate to each new set of input data to the computer. Suppose that there is an interval τ between each new input to the computer; then guidance signals are available at a frequency of $1/\tau$ and the error quantity may only be measured once in τ secs. This fact means that the system becomes, in effect, a sampling servo, and there is no guarantee that the suggested control system will be satisfactory in these circumstances.

A second effect which arises from the use of a digital computer is the delay introduced between measurement of the data and transmission of the appropriate signal to the missile; in effect the information is late by the amount of time taken to perform the computation. The effect of this delay has been examined both on the simulator and in theory.

II.2 Intermittent data - no delay

In the first instance, the effect on the missile performance of sampling the information was studied, on the assumption that the delay due to computation is negligible. It is probable that the data rate available from the computer will not be lower than 1 per second, and not higher than 5 per second, so that the simulator investigation consisted of examining the missile behaviour as the sampling period, τ, varied between $1/5$ and 1. It is necessary to maintain the loop closed between sampling instants by clamping the guidance signal to the value it had at the previous sampling instant. Thus if $\psi_C(t)$ is the output of the clamp and $\psi_\varepsilon(t)$ is the guidance signal,

$$\psi_C(t) = \psi_\varepsilon(n\tau) \qquad (n = 0,1,2....)$$

$$\text{for} \quad n\tau < t < (n+1)\tau \; .$$

The simulation with intermittent data has been done for a thrust-to-mass ratio of $\frac{1}{8}$g, rather than the $\frac{1}{4}$g used hitherto in this note, in line with more recent thoughts on the vernier stage. As is to be expected, after the inner loop has been appropriately adjusted, the lower available control acceleration roughly doubles the time taken to correct an initial error. Thus, when the error information is continuous, the time taken to correct a flight-path error of 3 mils is approximately 30 secs rather than 16 secs as obtained previously.

The same simulator arrangement was used as in the continuous case, but the loop was broken at X (Fig.8) and the sampling device inserted. The method used for simulation of the "sampling and clamping" is as described in Ref. 1; further details are given in Appendix IV. With the parameters chosen to give satisfactory performance in the continuous system, the missile response to a 3 mil step in ψ_T was examined for $\tau = 1/5$, $\tau = \frac{1}{2}$, and $\tau = 1$. It was found that the intermittent nature of the data tends to destabilize the system slightly, but even when τ is as large as 1 sec, the destabilizing effect is small. This has been confirmed by paper analysis using the pulse transfer function[1] which shows that instability is delayed until τ is greater than about 16 seconds (Appendix III).

II.3 Intermittent data with delay

The above simulation does not take account of the delay arising in the digital computer due to the finite time required for computation, and the simulator was therefore modified to include a delay of length τ, where $1/\tau$ is the sampling frequency. The method employed is described in detail in Appendix IV.

As is to be expected, the computing delay reduces the stability of the system. For $\tau = 1$ sec the system is markedly oscillatory but still stable; in Appendix III it is shown theoretically that instability occurs when τ is greater than about 7 secs.

In an attempt to regain the well-damped behaviour shown in the case of continuous data, the gain constant, K_1, was varied. It was found that for each sampling frequency, $1/\tau$, it is possible to choose an "optimum" value for K_1, – the word "optimum" being used to mean that for an initial step in ψ_T the time of settling is a minimum – and that the value of the "optimum" K_1 for a given τ is very nearly proportional to τ (Fig.17). Thus it appears that, provided a suitable loop gain is chosen, the performance of the suggested control system is only slightly degraded when the error information is sampled and delayed. Simulator records of responses to a 3 mil step in ψ_T are given in Figs. 18, 19.

II.4 Conclusions

1. Although sampling of the error data reduces the stability of the system, this reduction is very small for sampling periods of up to 1 sec.

2. The effect of a one period delay, τ, in the main feedback loop is to further reduce the stability; this effect is small for $\tau < \frac{1}{2}$ sec, but becomes marked at $\tau = 1$ sec.

3. The instability caused by the introduction of the delay can be compensated for, to a large extent, by reducing the loop gain, and the necessary reduction in loop gain is approximately proportional to the sampling period, i.e. to the delay.

4. The system suggested for azimuth control during the vernier stage is satisfactory even if the error data is sampled at a frequency of $1/\tau$ samples/sec and delayed by τ secs, provided that τ is not more than about 1 second.

APPENDIX III

Pulse transform analysis of system with intermittent data

(i) No delay

Following the method of Ref.1 we can split up the forward path of the loop into two filters, such that information passes from the first to the second filter only in samples.

We split up the clamp plus missile system into two components, the first filter representing an imaginary sampling device, and the second having the Laplace transform $p^{-1} Y_L(p)$, where $Y_L(p)$ is given by equation (29). [The justification for this step is given in the first appendix of ref.1.]

Then

$$Y_L(p) = \frac{K_1(w_o^2 - p^2)}{(1+T_1p)(p^2 + w_o^2 K_2 p + w_o^2 K_3)}$$

and so

$$p^{-1} Y_L(p) = \frac{K_1(w_o^2 - p^2)}{p(1+T_1p)(p^2 + w_o^2 K_2 p + w_o^2 K_3)}$$

$$= \frac{0.09375 - 0.1875p^2}{p^4 + 1.120p^3 + 0.5003p^2 + 0.000125p} .$$

This has the weighting function:-

$$W(t) = \mathcal{L}^{-1} \{p^{-1} Y_L(p)\}$$

i.e.

$$W(t) = 750 \left[1 - 0.99 e^{-\frac{t}{4000}} + e^{-0.56t} (0.416 \cos 0.42t - 0.316 \sin 0.42t) \right] .$$

Thus, the weighting sequence is:-

$$W(n\tau) = 750 \left[1 - 0.99 e^{-\frac{n\tau}{4000}} + e^{-0.56n\tau} (0.416 \cos 0.42n\tau - 0.316 \sin 0.42n\tau) \right] .$$

Now let the pulse transfer functions of the two filters be $U(z)$ and $V(z)$ respectively, so that the total loop transfer function, $W(z)$, is given by

$$W(z) = U(z) \cdot V(z) .$$

Then

$$V(z) = 750 \sum_{n=o}^{\infty} z^{-n} - 750 \sum_{n=o}^{\infty} \left(\frac{e^{-\frac{\tau}{4000}}}{z} \right)^n$$

$$+ \sum_{n=o}^{\infty} \left\{ \frac{e^{-0.56n\tau}(0.416 \cos 0.42n\tau - 0.316 \sin 0.42n\tau)}{z^n} \right\} .$$

Performing the summations, we obtain:-

$$V(z) = \frac{750.z}{z-1} - \frac{750.z\alpha}{z\alpha-1}$$

$$+ \frac{0.416(z\beta)^2 - z\beta(0.416C + 0.316S)}{(z\beta)^2 - 2z\beta C + 1}$$

where

$$\begin{cases} \alpha = e^{\frac{\tau}{4000}} \\ \beta = e^{0.56\tau} \\ C = \cos 0.42\tau \\ S = \sin 0.42\tau \end{cases} .$$

Then

$$W(z) = U(z) \cdot V(z)$$

$$= (1-z^{-1}) \cdot V(z) .$$

So

$$W(z) = \frac{750(\alpha-1)}{z\alpha-1} + (z-1) \left\{ \frac{0.416\beta^2 z - \beta(0.416C + 0.316S)}{(z\beta)^2 - 2z\beta C + 1} \right\} .$$

Now for stability of the closed-loop system, all the roots of the characteristic equation of the overall transfer function must lie inside the unit circle (Ref.1 - Appendix I).

The characteristic equation for the overall transfer function, $\frac{W(z)}{1+W(z)}$, is:-

$$z^3(\alpha\beta^2 + 0.4\alpha\beta^2)$$

$$+ z^2(-\beta^2 - 2\alpha\beta C + 750(\alpha-1)\beta^2 - 0.4\beta^2(1+\alpha) - \alpha\beta(0.416C + 0.316S))$$

$$+ z(\alpha + 2\beta C - 750(\alpha-1)\ 2\beta C + 0.4\beta^2 + (1+\alpha)\ \beta(0.416C + 0.316S))$$

$$+ (750(\alpha-1) - 1 - \beta(0.416C + 0.316S)) = 0 .$$

It is easily shown that to determine whether the roots of this equation lie inside the unit circle it is only necessary to transform the interior of the unit circle in the z-plane into the left half of the p-plane, using the transformation (for example):-

$$p = \left(\frac{z+1}{z-1}\right).$$

Then the Routh-Hurwitz criterion for roots in the left half plane can be applied to the transformed equation.

As an approximation we set $\alpha = 1$, except in terms containing $750(\alpha-1)$; then the coefficients of the transformed equation become:-

$$A_0 = 750(\alpha-1)\ [\beta^2 - 2C\beta + 1]$$

$$A_1 = [2 + 750(\alpha-1)]\beta^2 - [4C - 1500(\alpha-1)C]\beta + [2 - 2250(\alpha-1)]$$

$$A_2 = [5.6 - 750(\alpha-1)]\beta^2 - [4\gamma - 1500(\alpha-1)C]\beta - [4 - 2250(\alpha-1)]$$

$$A_3 = [3.6 - 750(\alpha-1)]\beta^2 + [4(\gamma+C) - 1500(\alpha-1)]\beta + [2 - 750(\alpha-1)]$$

where $\gamma = 0.416C + 0.316S.$

Substitution of trial values for τ leads to the condition for stability that:-

$\tau < \sim 16$ secs for stability.

(ii) Delay in forward path

It can be shown that when there is a delay of m periods in the error path, the overall pulse transform becomes

$$W_o(z) = \frac{z^{-m} . W(z)}{1 + z^{-m} . W(z)} .$$

Thus for a delay of one period, the characteristic equation becomes

$$z + W(z) = 0$$

and again the roots must lie within the unit circle for stability.

In detail, the equation becomes:-

$$z^4[\alpha\beta^2] + z^3\ [-\beta^2 - 2\alpha\beta C + 0.4\alpha\beta^2]$$
$$+ z^2\ [\alpha + 2\beta C + 750(1-\alpha)\beta^2 - 0.4\beta^2(1+\alpha) - \alpha\beta(0.416C + 0.316S)]$$
$$+ z\ [-1 - 750(\alpha-1)\ 2\beta C + 0.4\beta^2 + (1+\alpha)\beta\ (0.416C + 0.316S)]$$
$$+ [750(\alpha-1) - \beta(0.416C + 0.316S)] = 0 .$$

Evaluation of the coefficients of the corresponding transformed equation, and substitution of trial values of τ leads to the condition:-

<u>$\tau < \sim 7$ secs for stability.</u>

APPENDIX IV

Simulator details

IV.1 General

Most of the simulation was performed on a single-cabinet G-PAC computer, consisting of 20 drift-corrected D.C. amplifiers and built-in power supplies. The computing networks are mounted on plates which plug in to the amplifiers, so that computing impedances are readily changed if necessary. Interconnections are made from the fronts of the computing plates and signal bus-bars are available at the side of the cabinet, as are the computing voltages (± 100V) and step voltages (± 100 V). Facilities are available for introducing initial conditions onto the amplifiers when used as integrators.

IV.2 The sampling device

The simulation of the sampling device is basically that used in Ref.1. An asymmetric multivibrator drives a high speed relay whose moving contact carries the error signal (Fig.20); for most of the multivibrator period (the "space") the moving arm makes with an isolated fixed contact, but during the relatively short "mark" period contact is made with the other fixed contact which is connected via a small resistor to the storage capacitor (clamp). (The purpose of the small resistor is to ensure that the cathode follower of the previous amplifier is not called upon to supply too much current).

Since the relay makes contact for a finite time - approximately the "mark" period of the multivibrator - the arrangement is not a true sampling device, i.e. its output is not in the form of δ-functions. However, provided that the mark-space ratio of the multivibrator is less than, say, 0.05, a good approximation to a sampling device is obtained.

IV.3 The delay mechanism

The production of a delay equal to one sampling period is obtained by using an extension of the sampling simulator previously described. An asymmetric multivibrator drives a Type 3000 heavy-duty relay; during the short "mark" period this relay closes a circuit putting 24V on the coil of a standard four-bank double-arm uniselector. The input voltage (error signal) is connected to the first wiper and the output lead is connected to the second wiper; a storage capacitor is connected (through a small resistor) to the first contact of bank 1 and the output of this condenser is taken to the third contact of bank 2; similarly a second storage condenser is connected between contact 2 of bank 1 and contact 4 of bank 2, and so on. In this case five storage condensers were used, and every fifth contact on each bank was commoned. The mode of operation is as follows. Suppose that all the capacitors have no charge on them and that at $t = 0$ the wipers are on contact 1; then at $t = \tau$ the wipers move on to contact 2 and the error signal is applied to contact 2, bank 1. The output (i.e. the voltage on contact 2, bank 2) is zero, since this is the voltage on the capacitor whose input is contact 50, bank 1 (≡ contact 5, bank 1). Now at $t = 2\tau$ the wipers move on to contact 3 and the voltage left on contact 2, bank 1, is that appropriate to the error signal at $t = 2\tau$ and <u>not</u> $t = \tau$. The voltage picked off contact 3, bank 2, is the output of the capacitor whose input is contact 1, bank 1; i.e. the output at $t = 2\tau$ is the error voltage appropriate to $t = \tau$. Then at $t = 3\tau$ the wipers move to contact 4 and in this case the output wiper picks off the voltage from the capacitor whose input is contact 2, bank 1. This voltage is that corresponding to the error at $t = 2\tau$ (see above) and so a delay between input and output, of length τ, is obtained.

In final form, the sampling unit and delaying unit are combined into one unit, and a switching arrangement is provided so that the output of the unit may be sampled but not delayed, or sampled and delayed by one sample period.

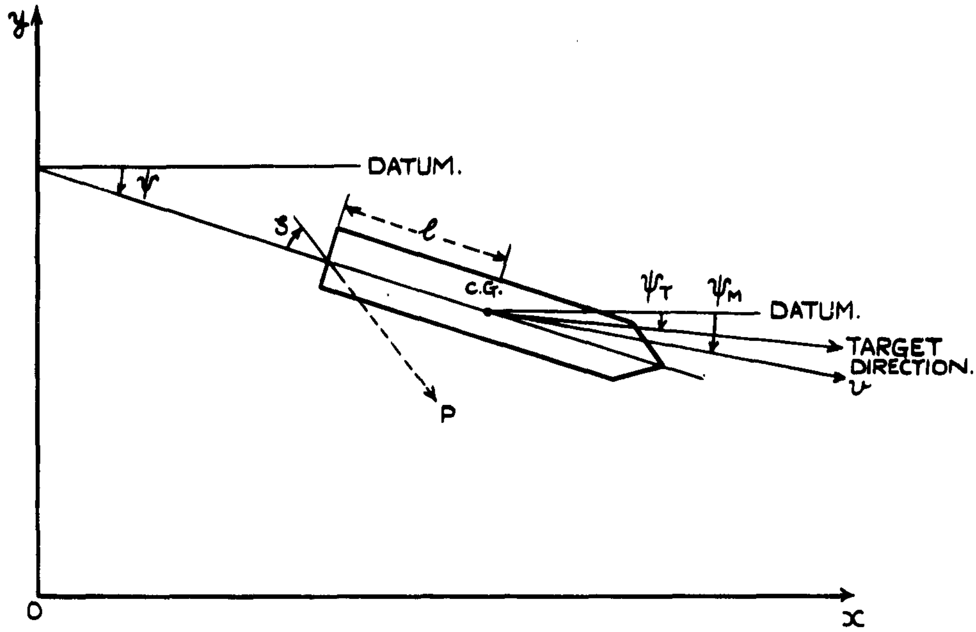

FIG. 1. ANGLES IN THE AZIMUTH PLANE.

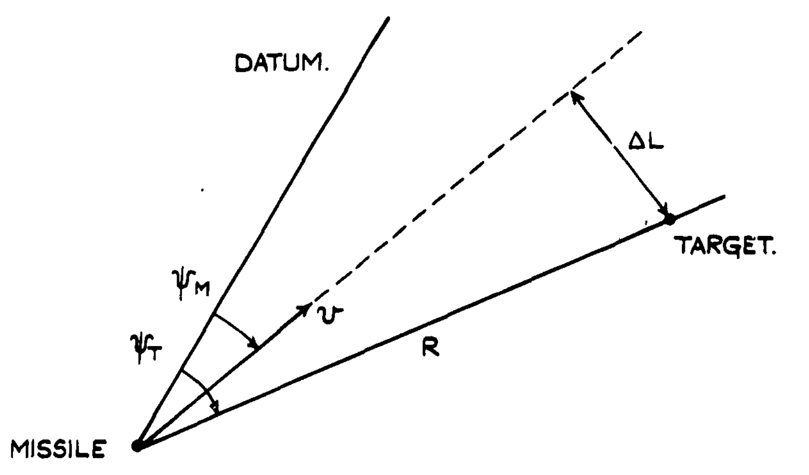

FIG. 2. RELATION BETWEEN LINE AND ANGULAR ERRORS.

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T.N. G.W. 394.

FIG.3-5.

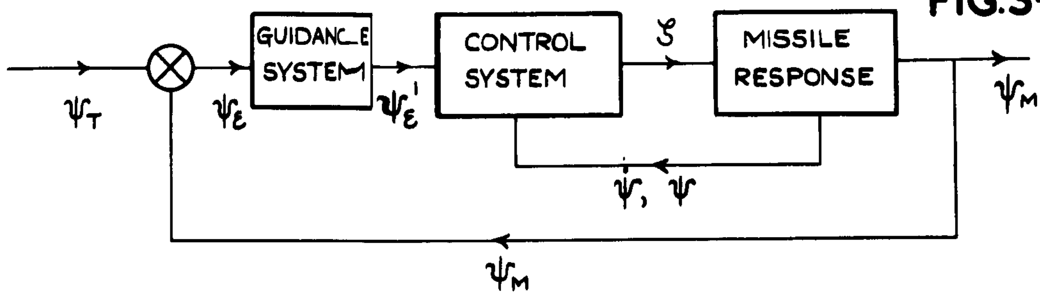

FIG.3. BLOCK DIAGRAM OF COMPLETE SYSTEM.

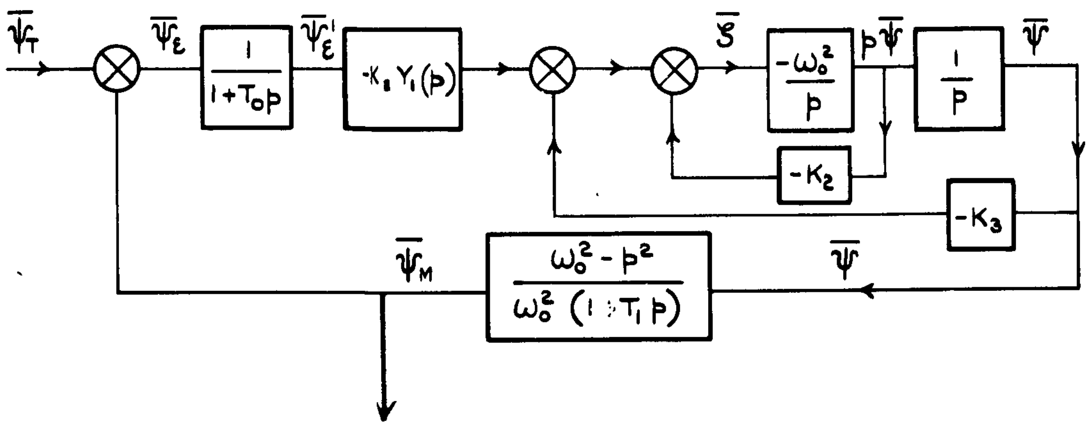

FIG.4. REPRESENTATION OF LINEARIZED EQUATIONS.

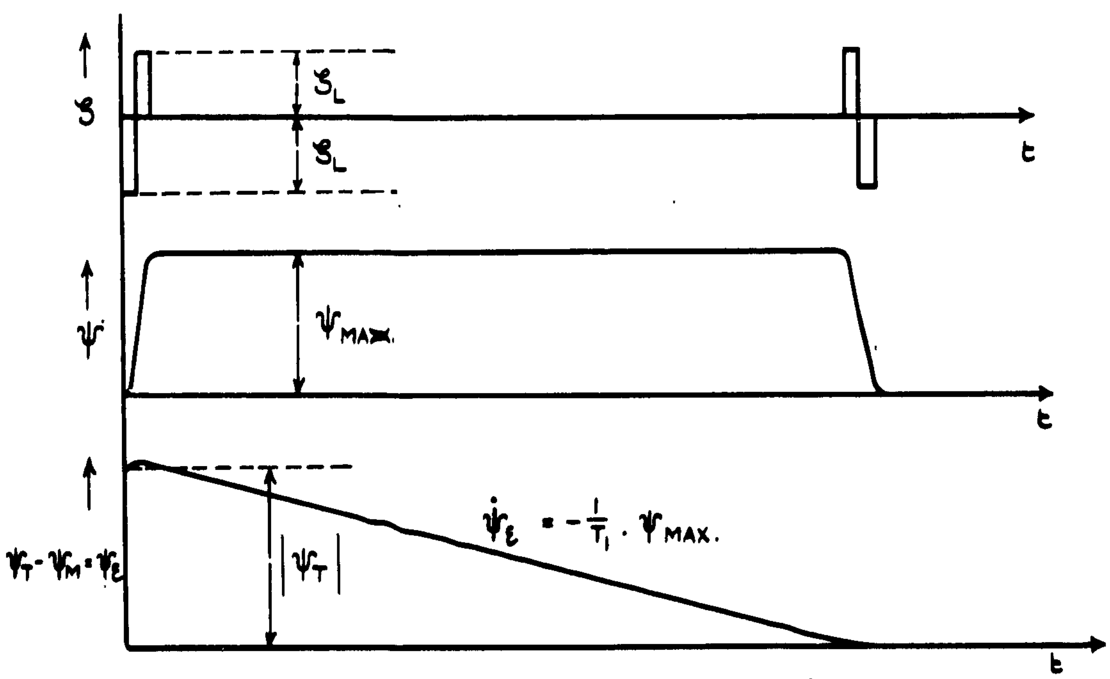

FIG.5. RESPONSE TO A STEP IN ψ_T FOR A NON-LINEAR SYSTEM WITH VERY LOW INERTIA.

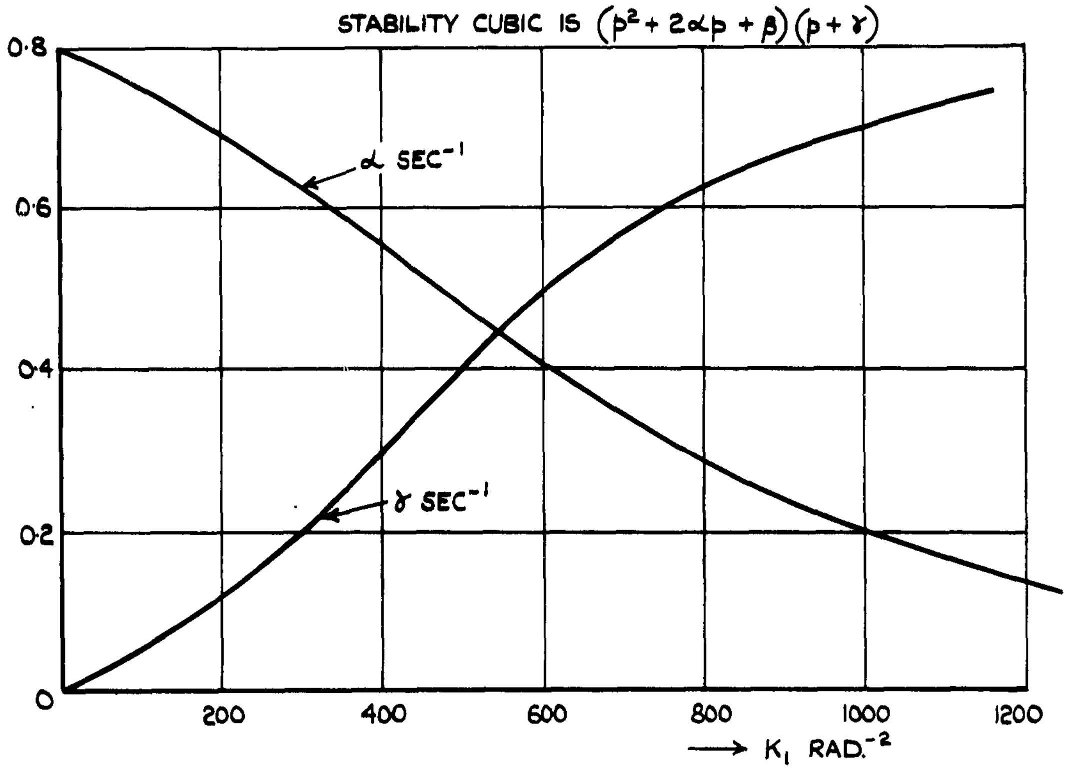

FIG.6. EFFECT OF CHANGE OF GAIN ON THE DAMPING DISTRIBUTION.

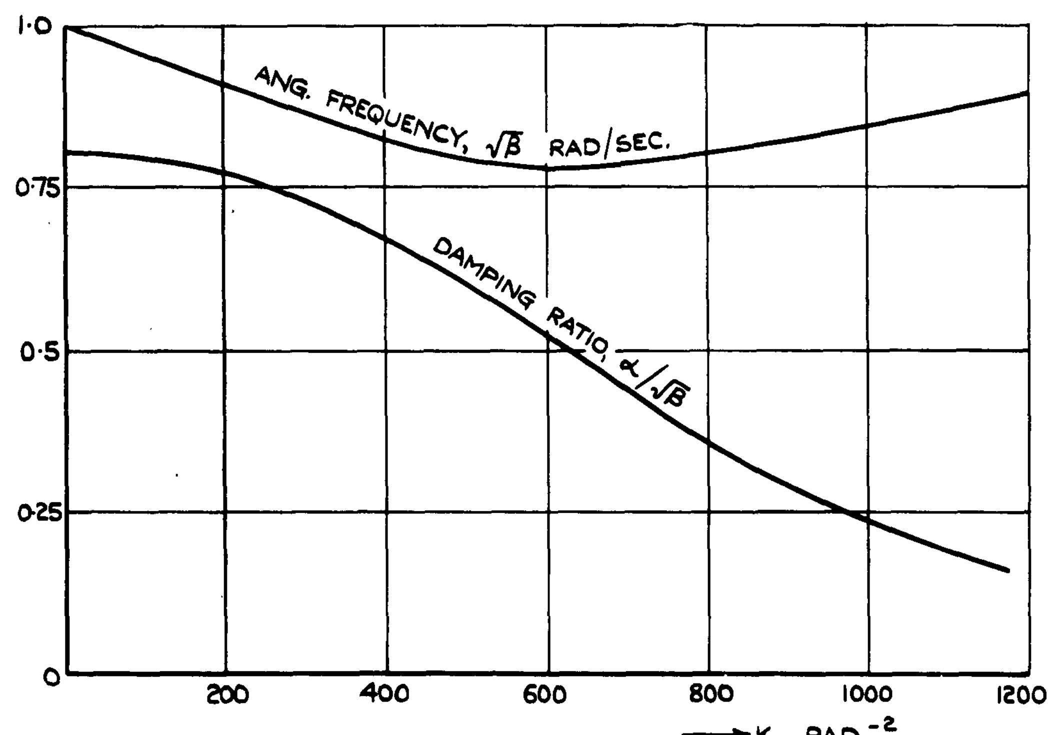

FIG.7. EFFECT OF CHANGE OF GAIN ON THE FREQUENCY AND DAMPING RATIO.

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T.N. G.W.394.
FIG. 8.

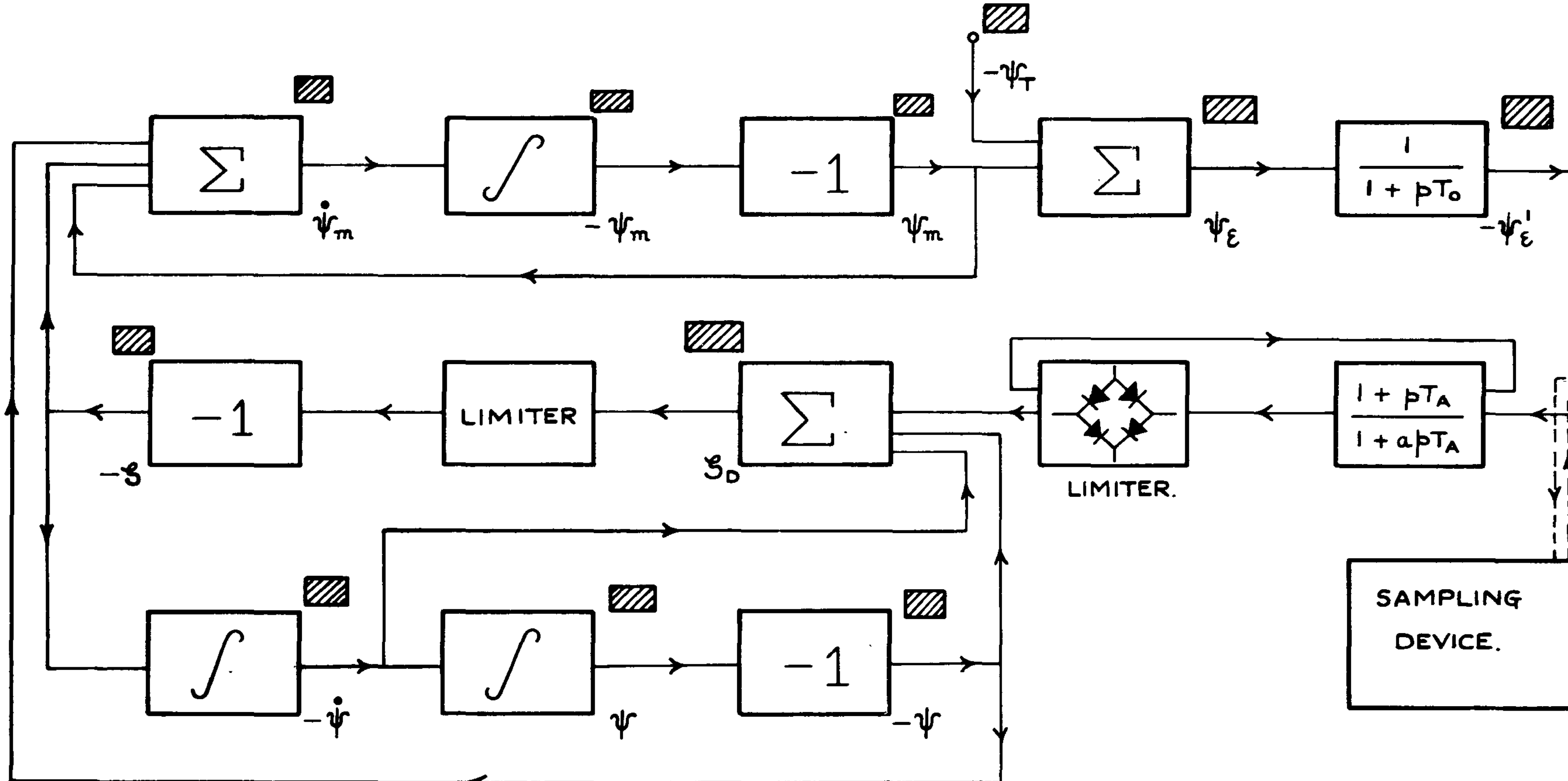


FIG. 8. BLOCK SCHEMATIC OF SIMULATOR ARRANGEMENT.

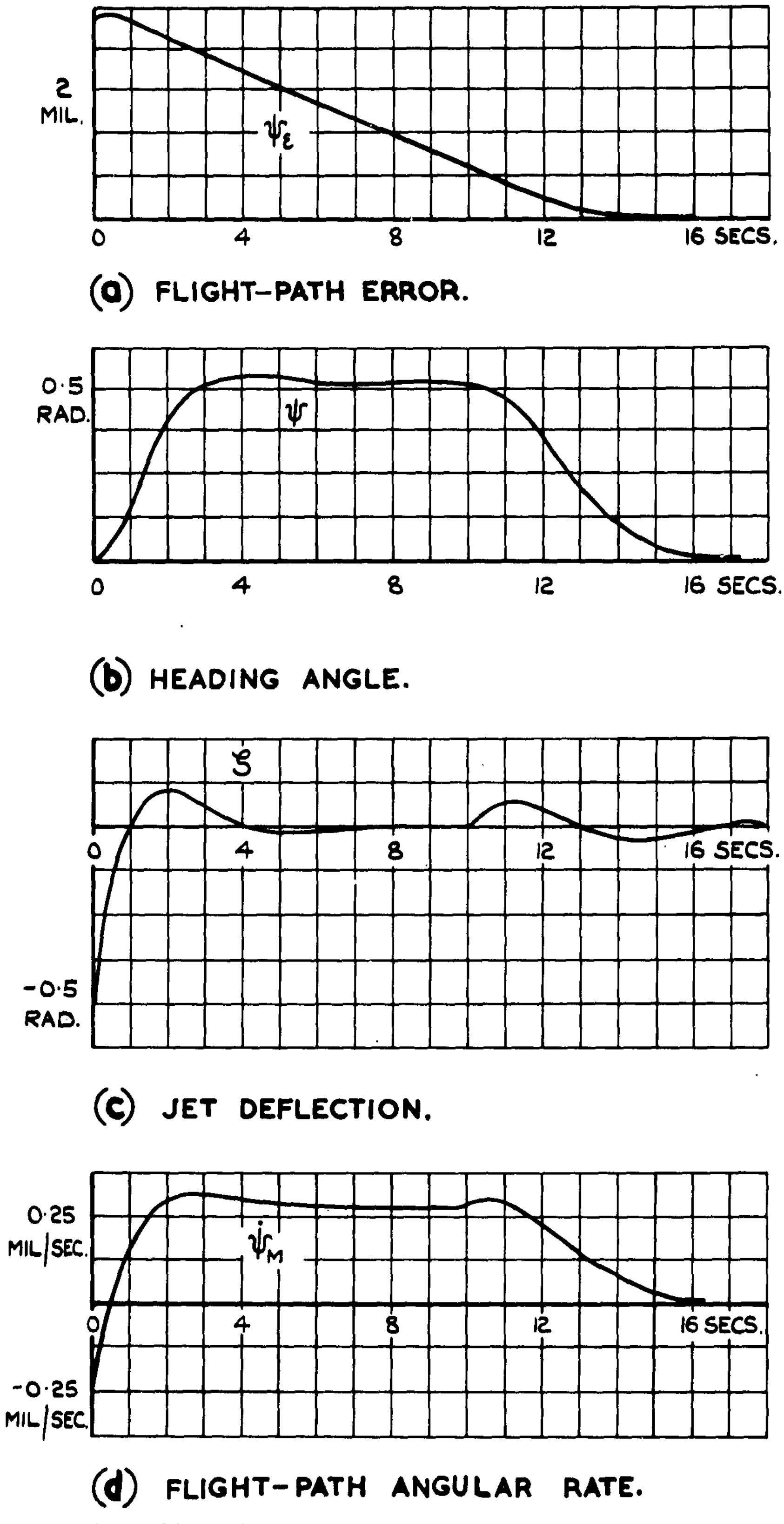


FIG.9. (a-d) SIMULATOR RECORDS — RESPONSES FOR STEP DEMAND OF ψ_T (·003 RAD.) $K_1 = 550$, $K_2 = 1{\cdot}6$, $K_3 = 1$, NO PHASE ADVANCE $\psi_0 = \dot{\psi}_0 = 0$.

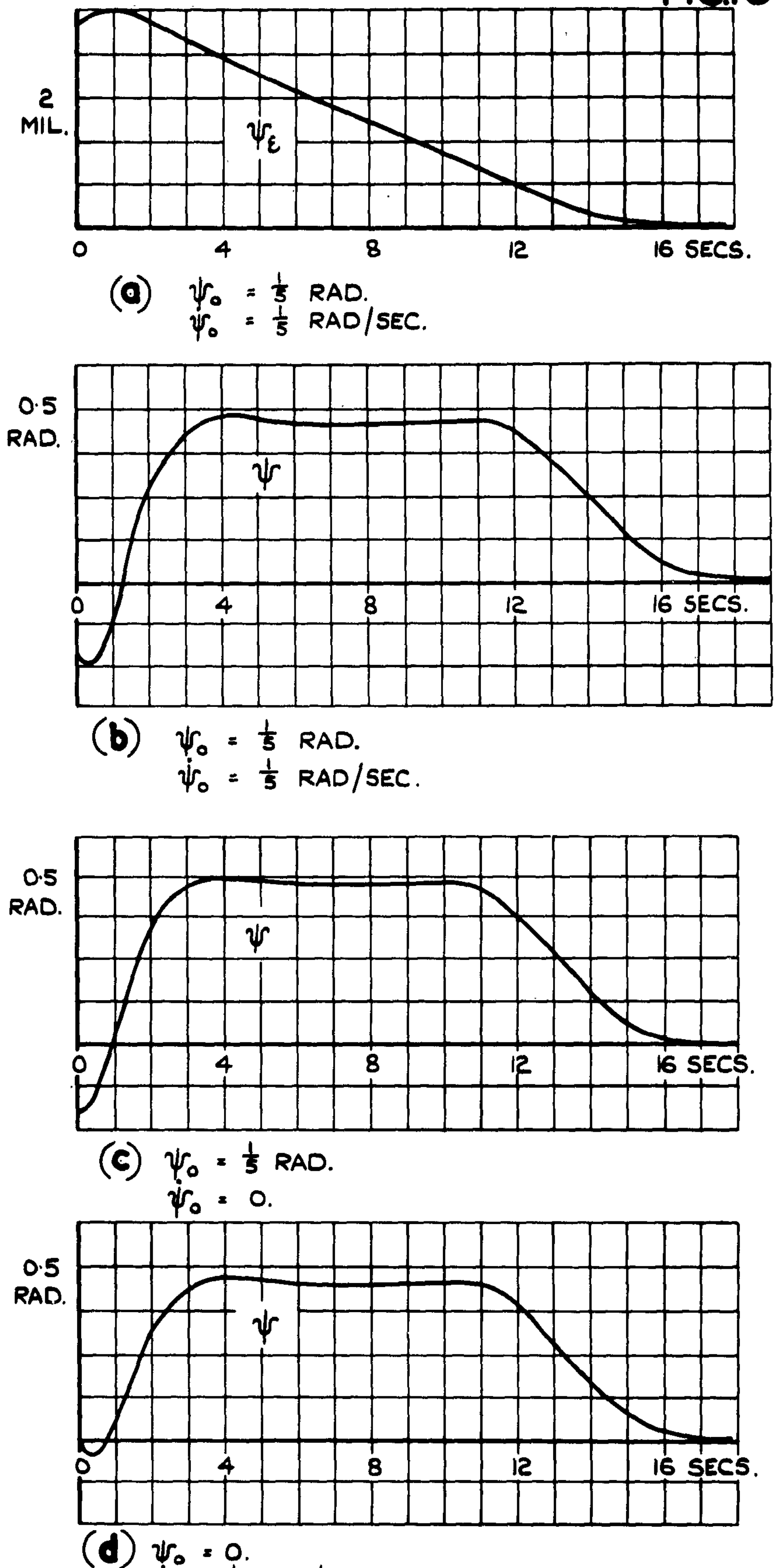


FIG.10 (a-d) EFFECT OF INITIAL CONDITIONS.
CONTROL PARAMETERS AS IN FIG.9.

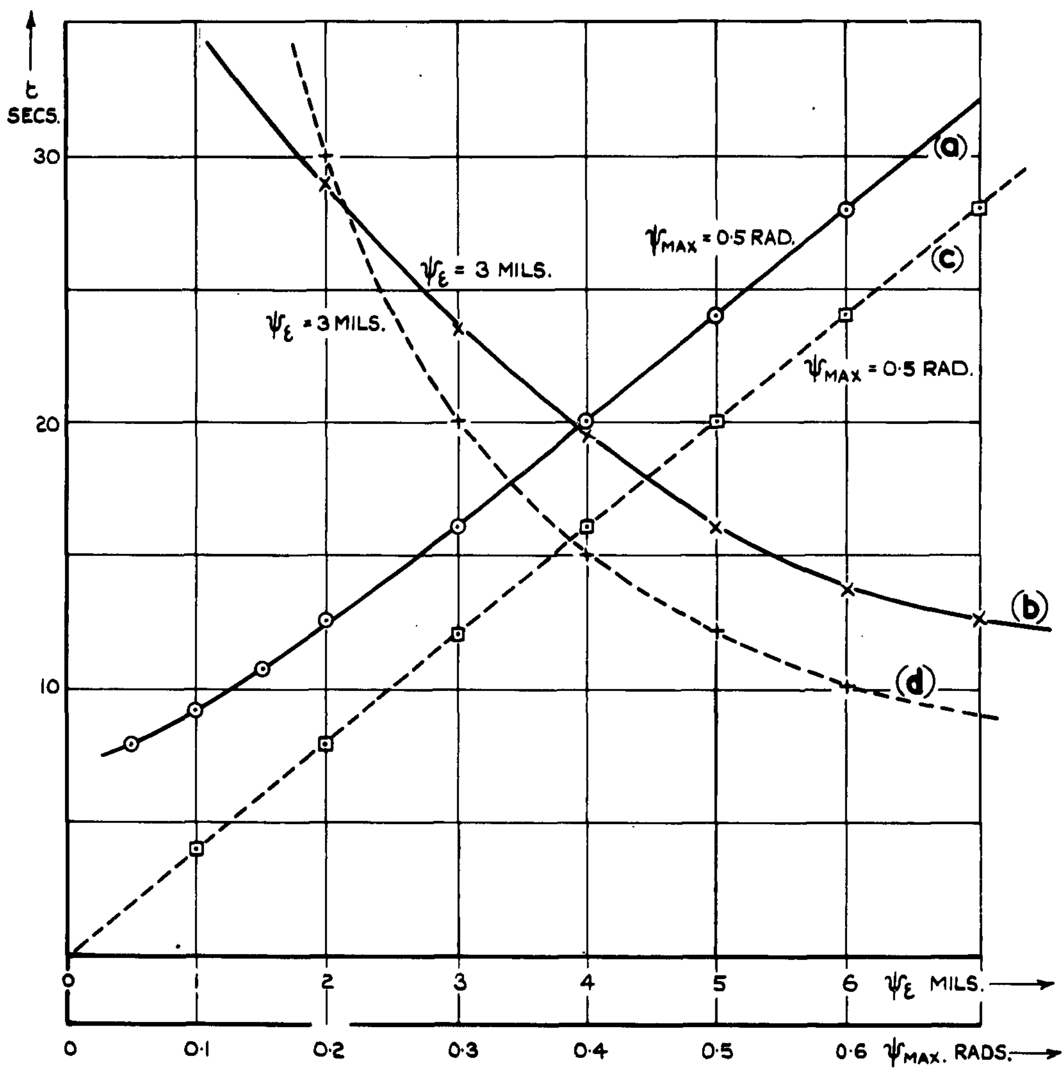


FIG.11. TIME TAKEN TO REDUCE INITIAL ERROR, PLOTTED.

(a) AS A FUNCTION OF ψ_ε (WITH $\psi_{MAX} = 0{\cdot}5$ RAD.)
(b) AS A FUNCTION OF ψ_{MAX} (WITH $\psi_\varepsilon = 3$ MILS.)
(c) CURVE (a) FOR THE IDEAL SYSTEM
(d) CURVE (b) FOR THE IDEAL SYSTEM
} WITH $T_1 = 2000$ SECS.

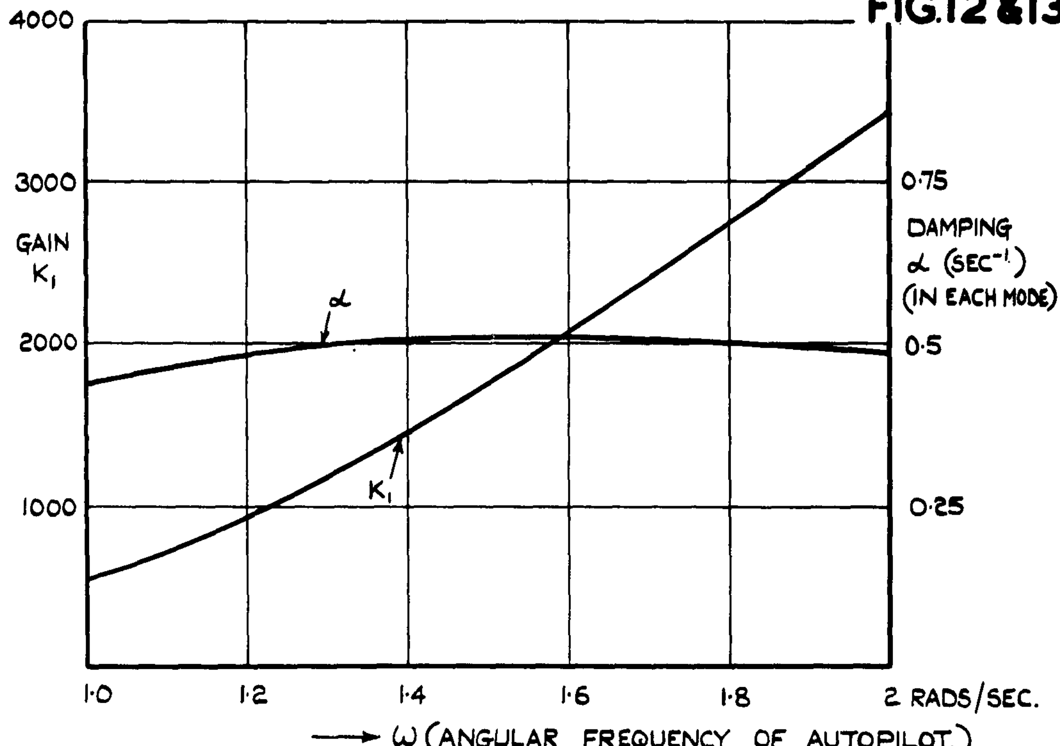


FIG.12. EFFECT OF INNER LOOP GAIN ON GAIN REQUIRED TO GIVE EVEN DAMPING DISTRIBUTION.

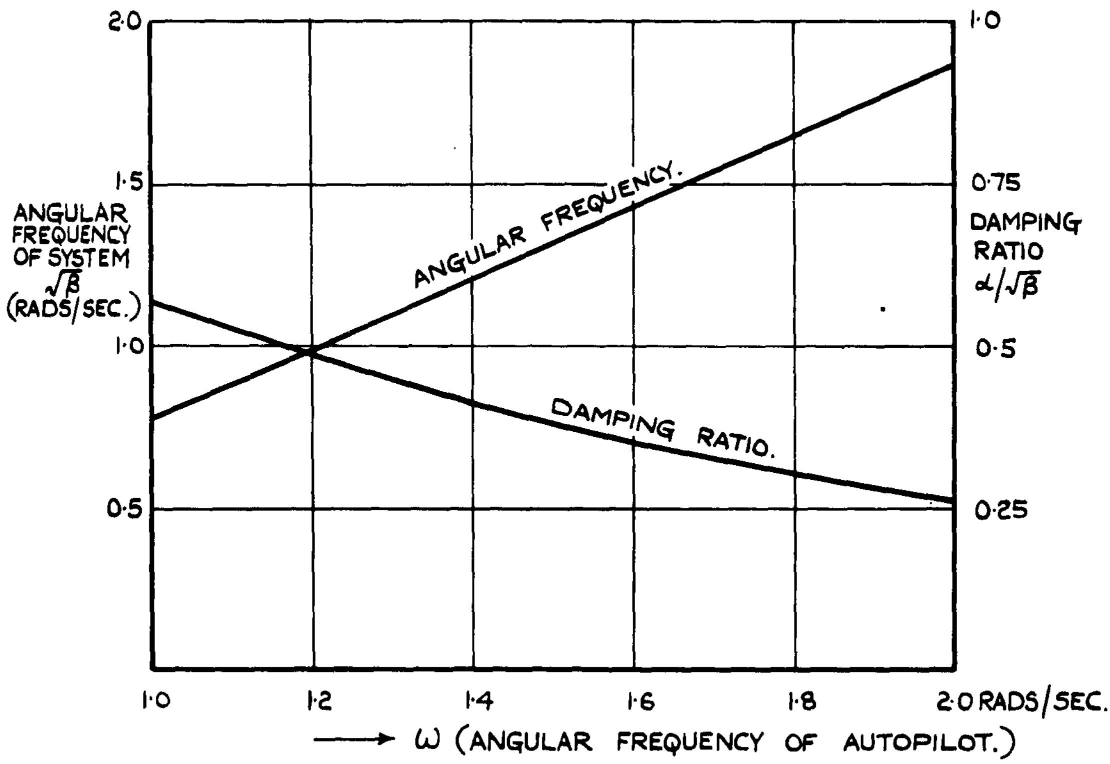


FIG.13. EFFECT OF INNER LOOP GAIN ON SYSTEM FREQUENCY AND DAMPING RATIO WITH THE GAIN CHOSEN TO GIVE EVEN DISTRIBUTION OF DAMPING.

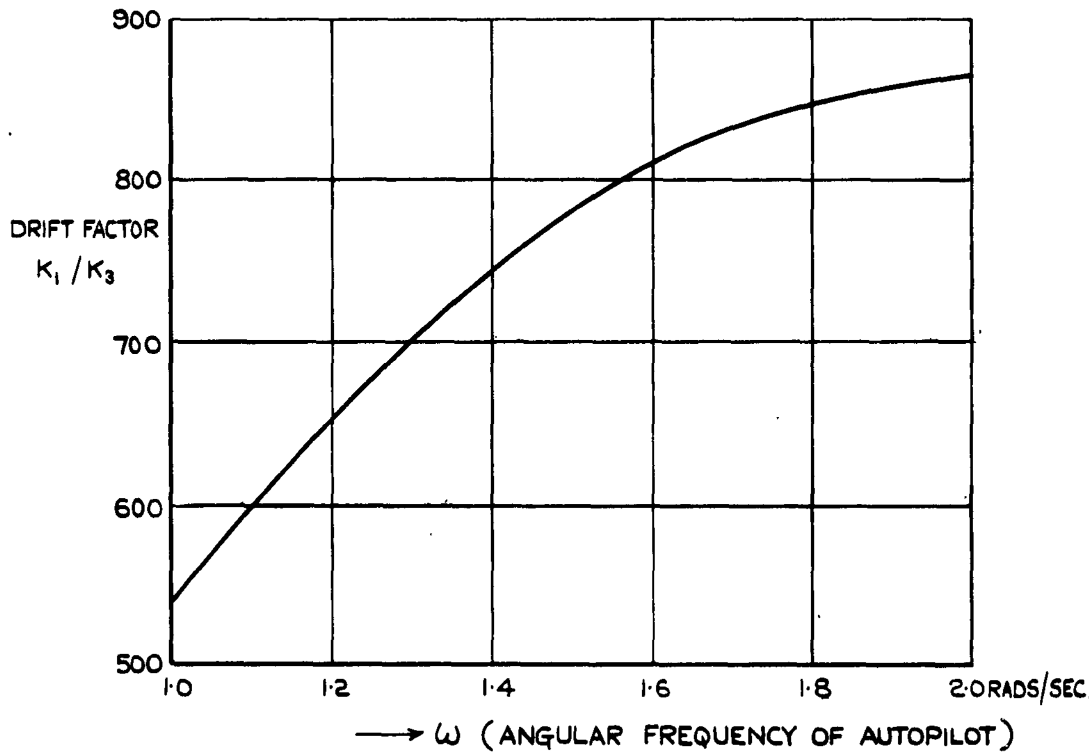
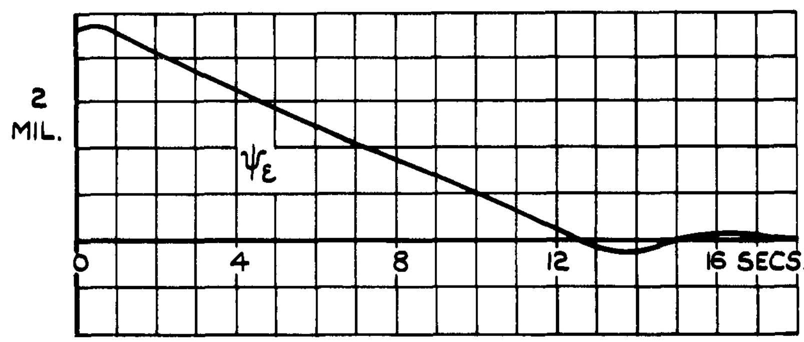
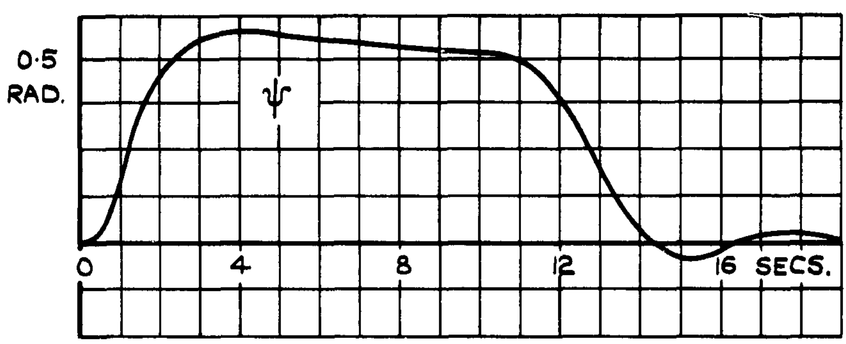


FIG. 14. EFFECT OF INNER LOOP GAIN ON THE DRIFT FACTOR K_1/K_3 WHEN THE DAMPING IS EVENLY DISTRIBUTED BETWEEN THE MODES.



(a) FLIGHT-PATH ERROR.



(b) HEADING.

FIG.15 (a & b) RESPONSES WITH A HIGHER INNER LOOP GAIN.
$K_1 = 2{,}800,\ K_2 = 3{\cdot}46,\ K_3 = 3.$
NO PHASE ADVANCE.

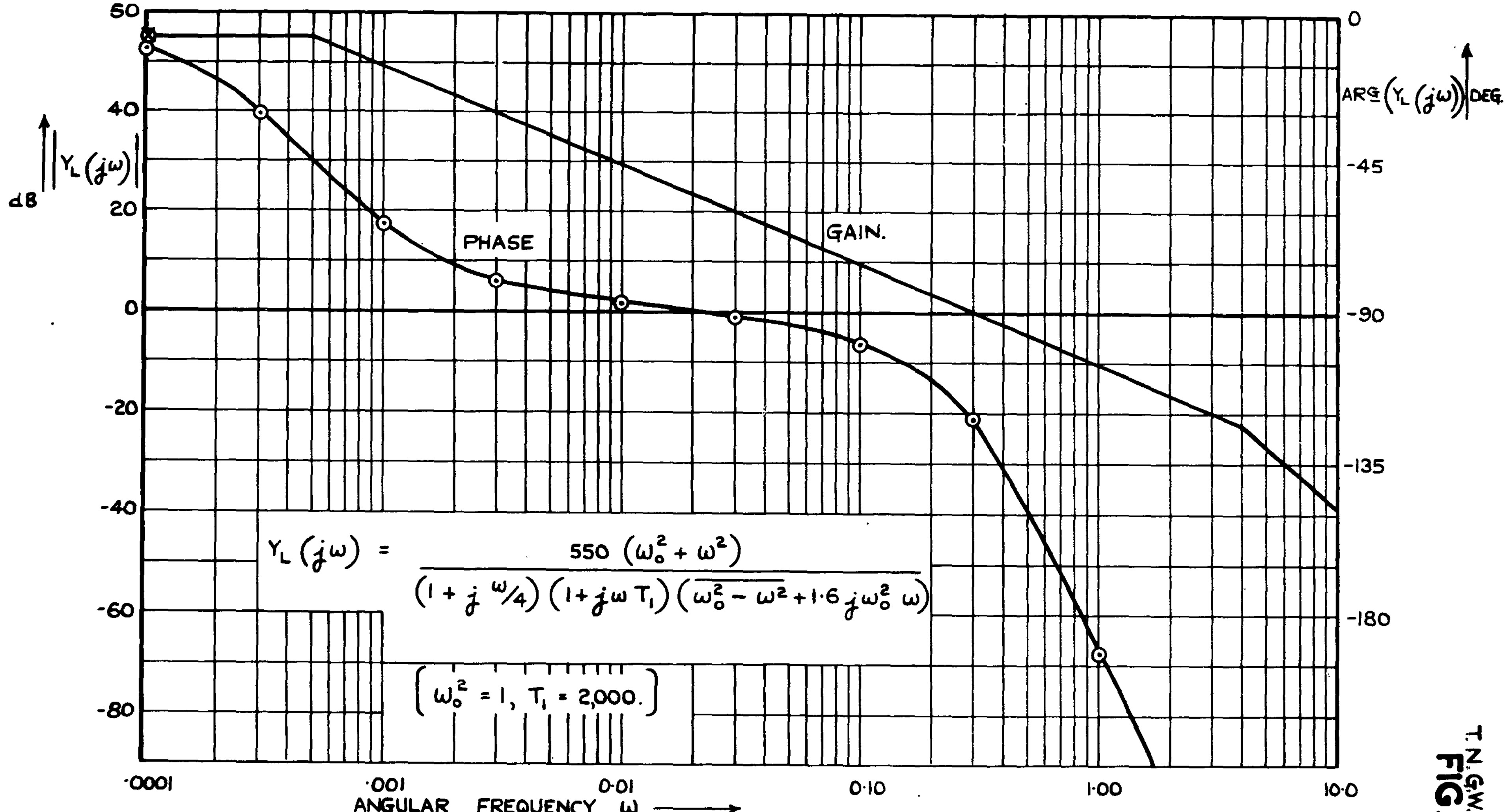


FIG.16. GAIN-AND PHASE-LOG FREQUENCY CURVES FOR $Y_L(j\omega)$

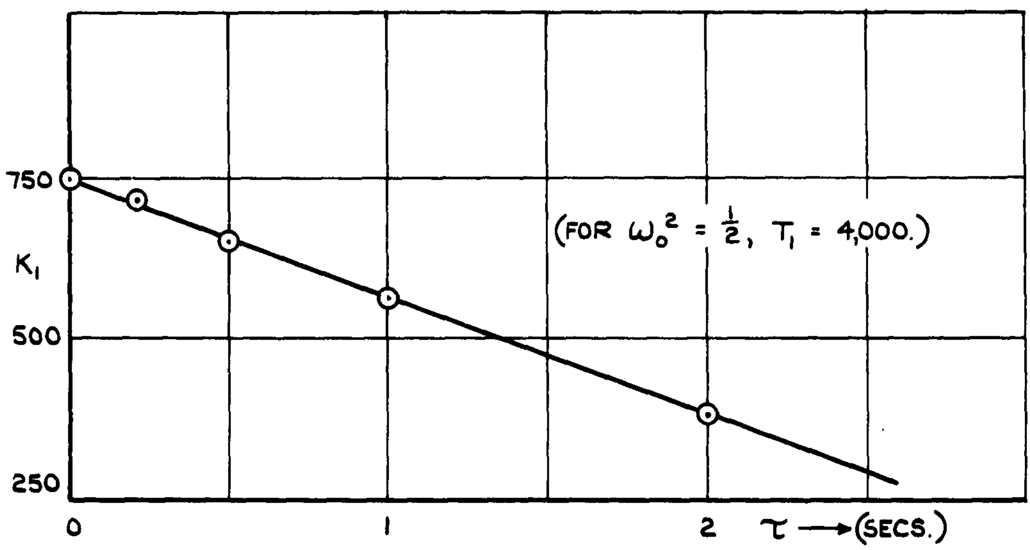


FIG.17. VARIATION OF "OPTIMUM" GAIN WITH SAMPLING INTERVAL AND DELAY TIME OF τ.

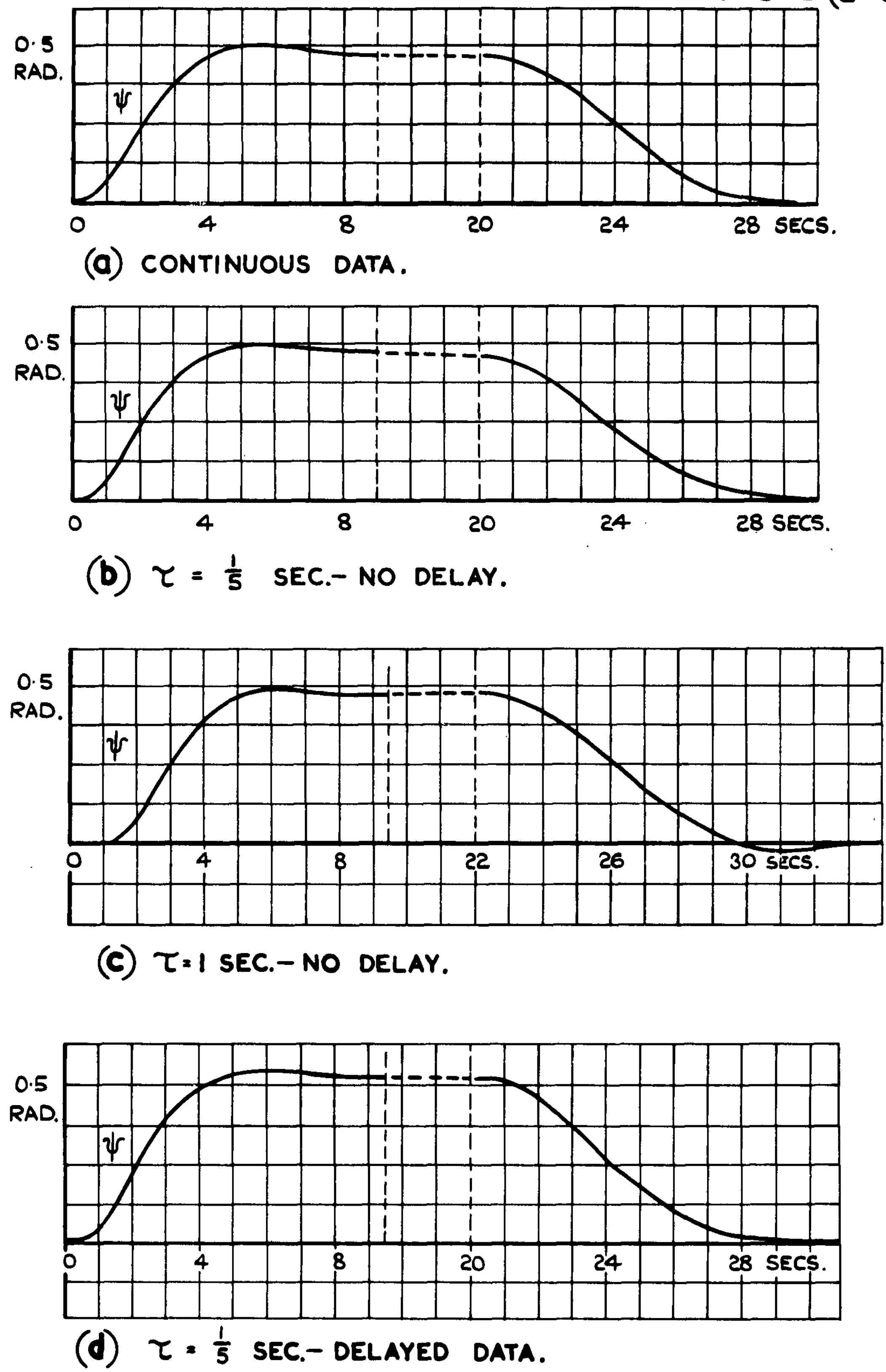
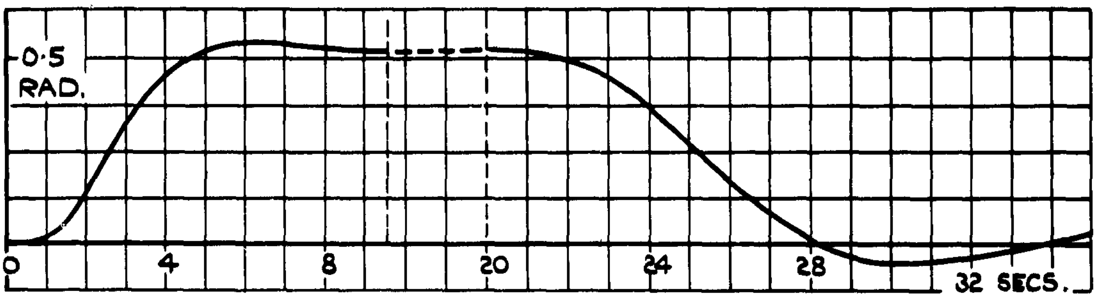
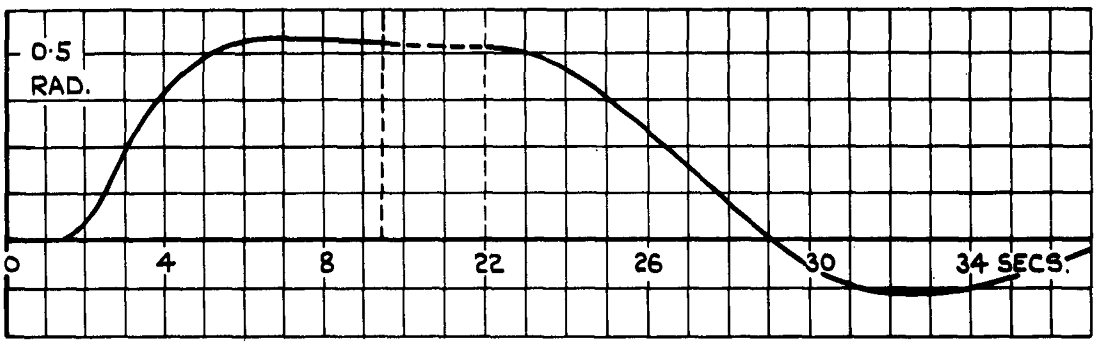


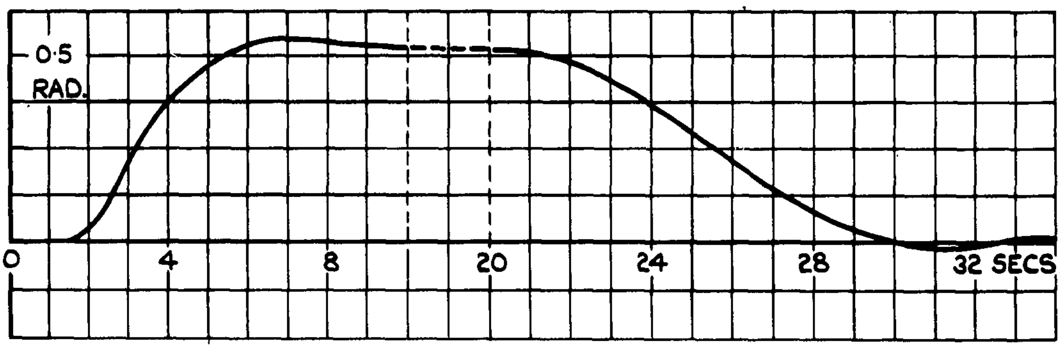
FIG. 18.(a-d) COMPARISON OF HEADING RECORDS ($\dot{v} = \frac{1}{8} g$)



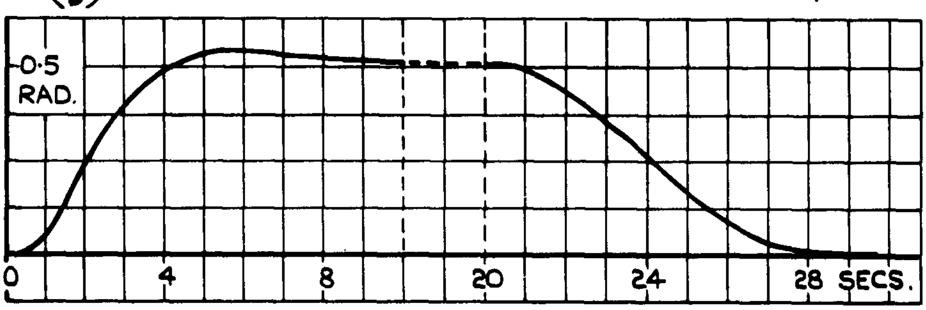
(e) $\tau = \frac{1}{2}$ SEC. – DELAYED DATA.



(f) $\tau =$ 1 SEC. – DELAYED DATA.



(g) $\tau =$ 1 SEC. – DELAYED DATA – "OPTIMUM" K_1.



(h) $\tau = \frac{1}{2}$ SEC. – DELAYED DATA – "OPTIMUM" K_1.

FIG.19 (e–h) COMPARISON OF HEADING RECORDS
(CONTINUED)
($\dot{v} = \frac{1}{8}g$)

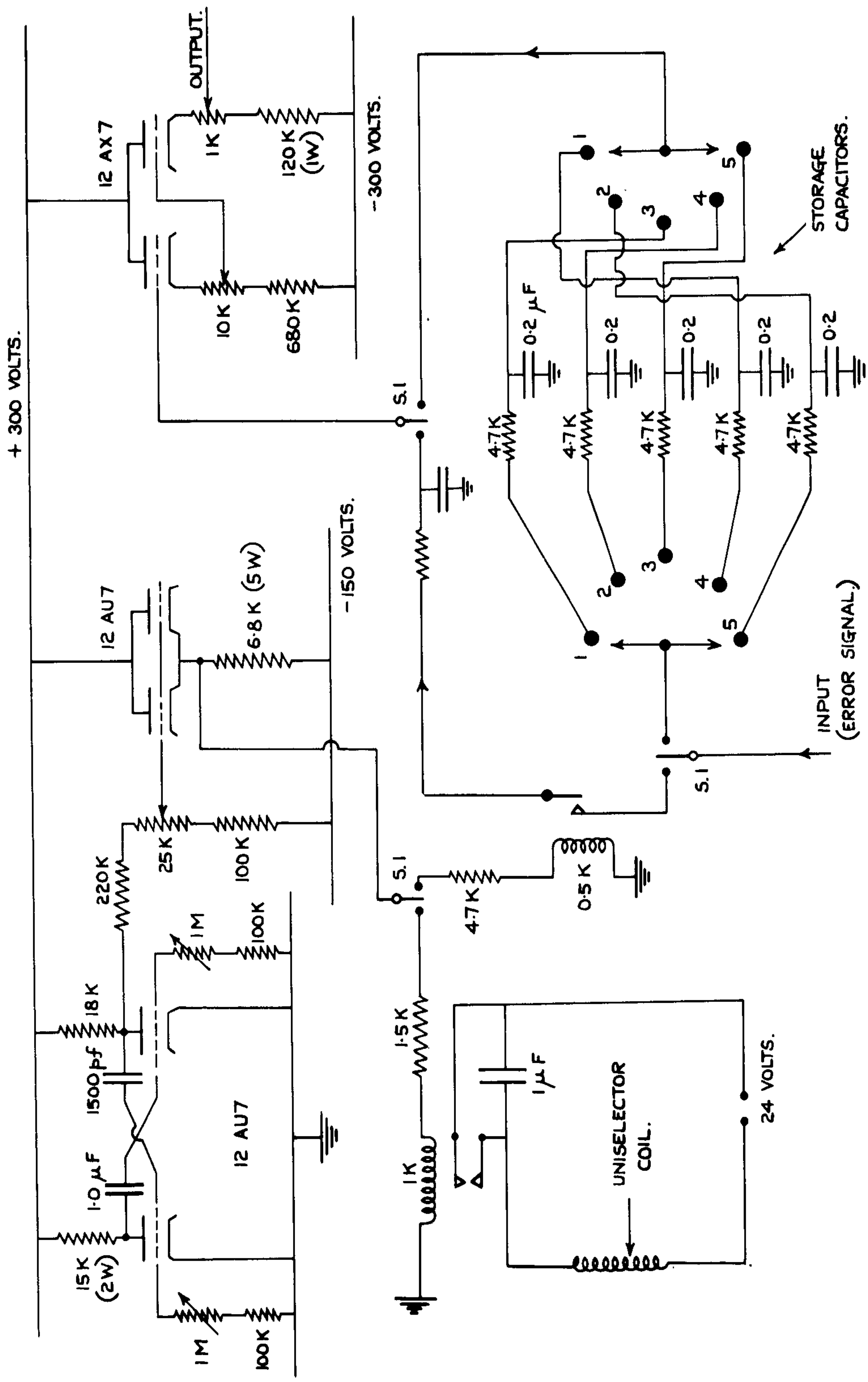


FIG.20. CIRCUIT OF SAMPLING DEVICE.

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1956.1
Merson, R. H. and Howl, J. M.

A STUDY OF AZIMUTH GUIDANCE AND CONTROL FOR THE VERNIER STAGE OF BLUE STREAK

A control system in azimuth based on a small swivelling jet is proposed for the vernier stage of the medium range ballistic missile Blue Streak. This control system is applicable to both inertia and radar guidance systems, the measured error quantity being essentially the azimuth component of the deviation of the missile flight path from the line joining missile and target.

The jet has to be sufficiently small to avoid a missile speed error at cut-off. This leads to a relatively small available deflecting force in azimuth and, to make full use of this, a system with a high loop gain and limited error input is proposed, stabilization being obtained with heading and heading rate feedback. The input limits and gains are chosen so that, in effect, the missile heading is limited within prescribed bounds.

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The performance of the system has been studied on an analogue simulator under assumptions of constant missile parameters and the usual small angle approximations. It is shown that, with proper choice of control coefficients, the system is very little worse than an ideal, infinitely stiff, system with the same heading limitation.

The steady state error due to heading gyro wander is considered, and for the missile parameters used is thought to be tolerable.

The effects of the data sampling and computer time lag relevant to the radar guidance system have been studied and it is shown that sampling periods and time lags up to about 1 second have no material effect on the control system.

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